CORRECTION OF THE ERROR IN DETERMINING THERMOPHYSICAL CHARACTERISTICS DUE TO THE HEAT CAPACITY OF THE HEATER

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UDC 536.2.083

The article presents an approach to the calculation of the systematic error, introduced by the heat capacity of the heater itself, in methods using a circular heat source of constant power for determining thermophysical characteristics.

Methods with a circular heat source of constant power for determining thermophysical characteristics have been widely in use lately [1-4]. They are based on solving problems of thermal conductivity on the assumption that the heat capacity of the heater is equal to zero. In practice this condition is not fulfilled, and therefore the heat capacity of the heater has a distorting effect on the formation of a nonsteady temperature field in the specimen.

Since in the practical realization heaters are made of thin (0.1-0.15 mm diameter) manganin or constantan wire whose thermal conductivity is more than one order of magnitude higher than that of the investigated materials, the temperature gradient across the thickness of the heater (and consequently also the thickness of the heater) may be neglected, but its surface heat capacity c_h has to be taken into account.

Let us examine the following problem: on the surface of the investigated (thermally) semibounded body there is a thin circular heater situated, with radius r_0 and surface heat capacity c_h. At the instant $\tau = 0$ the constant specific power w_0 is supplied to it. Beyond the heater there is no heat exchange.

The mathematical statement of the problem is:

$$\frac{\partial T}{\partial \tau} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right),\ \tau > 0; \tag{1}$$

$$T(r,\ z,\ 0) = T_0; \tag{2}$$

$$-\lambda\frac{\partial T(r,\ 0,\ \tau)}{\partial z} = w_0 - c_h\frac{\partial T}{\partial \tau},\ r \leqslant r_0,\ z = 0; \tag{3}$$

$$\frac{\partial T(r,\ 0,\ \tau)}{\partial z} = 0,\ r > r_0; \tag{4}$$

$$\frac{\partial T(\infty,\ z,\ \tau)}{\partial r} = \frac{\partial T(r,\ \infty,\ \tau)}{\partial z} = \frac{\partial T(0,\ z,\ \tau)}{\partial r} = 0. \tag{5}$$

The solution of Eq. (1) with the initial and boundary conditions (2)-(5), obtained by operational methods, can be written for the center of the heater (points z = 0, r = 0) in the form

$$\Delta T(\tau) = T(0,\ 0,\ \tau) - T_0 = \tag{6}$$
$$= \frac{w_0}{2\pi i}\int\limits_{\sigma-i\infty}^{\sigma+i\infty}\frac{\left[1-\exp\left(-\sqrt{s}\,\frac{r_0}{\sqrt{a}}\right)\right]\exp(s\tau)\,ds}{bs\sqrt{s} + c_h s^2\left[1-\exp\left(-\sqrt{s}\,\frac{r_0}{\sqrt{a}}\right)\right]}.$$

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 0022-0841/86/5006-0724$12.50

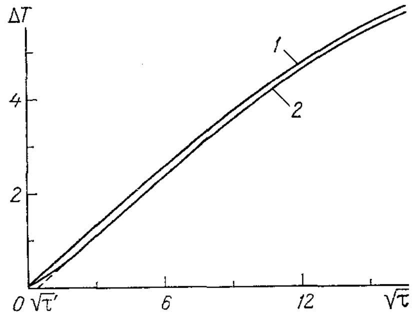

Fig. 1

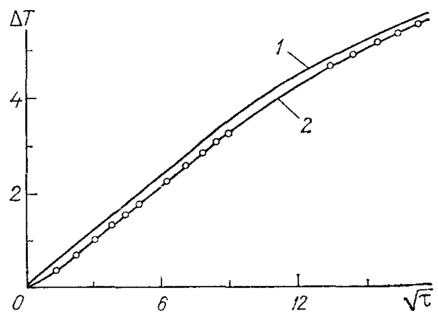

Fig. 2

Fig. 1. Dependence of the change of excess temperature ΔT (°K) of the center of a circular heat source without (curve 1) and with a view (curve 2) to the heat capacity of the heater itself on $\sqrt{\tau}$ ($\sec^{1/2}$) for rubber ($a = 0.138 \cdot 10^{-6}$ m²/sec, $\lambda = 0.05$ W/(m·°K), $w_0 = 50$ W/m², $r_0 = 10 \cdot 10^{-3}$ m, $c_h = 50$ J/(m²·°K)).

Fig. 2. Theoretical dependences of the change of excess temperature ΔT (°K) of the center of the circular heat source disregarding (curve 1) and taking into account (curve 2) the proper heat capacity of the heater on $\sqrt{\tau}$ ($\sec^{1/2}$) and the point of the experimental temperature curve for polymethyl methacrylate ($a = 1.28 \cdot 10^{-7}$ m²/sec, $\lambda = 0.21$ W/(m·°K), $c_h = 398$ J/(m²·°K), $w_0 = 205.4$ W/m², $r_0 = 9.9 \cdot 10^{-3}$ m).

In solving (6), contour integration with respect to the complex variable s is carried out along the straight line Re s = σ.

Finding the inverse Laplace transform poses considerable difficulties, expression (6) was therefore numerically inverted. For that we used an algorithm presented in [5]:

$$F_a = \frac{\ln 2}{\tau} \sum_{i=1}^{N} V_i P\left(\frac{\ln 2}{\tau}\, i\right),$$

$$V_i = (-1)^{\frac{N}{2}+i} \sum_{k=\left[\frac{i+1}{2}\right]}^{\min\left(i,\, \frac{N}{2}\right)} \frac{k^{\frac{N}{2}}\,(2k)!}{\left(\frac{N}{2}-k\right)!\,k!\,(k-1)!\,(i-k)!\,(2k-i)!},$$

where F_a is the approximate magnitude of the original F (τ); P (s) is the Laplace transform.

In [5] it was noted that the accuracy of calculations depends on the number N. For the ordinary computer accuracy (the calculations were carried out on an ES1020) the smallest error of the values of the original is attained for N = 10. A check of the operation of the algorithm showed that the error of inversion does not exceed ±3% for the class of transforms analogous to (6) (in particular, we inverted transforms of the form $P(s) = a[1-\exp(-k\sqrt{s})/s\sqrt{s}]$.

To improve the accuracy of inversion, we transform the initial Laplace transform to the form

$$\Delta T(s) = \frac{w_0\left[1-\exp\left(-\sqrt{s}\,\frac{r_0}{\sqrt{a}}\right)\right]}{bs\sqrt{s}} - \frac{c_h w_0\left[1-\exp\left(-\sqrt{s}\,\frac{r_0}{\sqrt{a}}\right)\right]^2}{sb^2 + c_h bs\sqrt{s}\left[1-\exp\left(-\sqrt{s}\,\frac{r_0}{\sqrt{a}}\right)\right]}. \tag{7}$$

In expression (7) the first term represents the temperature of the center of the heating spot when $c_h = 0$, and the original of this transform is known [6]:

$$\Delta T_1(\tau) = \frac{2w_0\sqrt{\tau}}{b}\left(\frac{1}{\sqrt{\pi}} - \operatorname{ierfc}\frac{r_0}{2\sqrt{a\tau}}\right). \tag{8}$$

The second term takes the effect of the heat capacity of the heater on the formation of the temperature into account.

Calculations show that even when heaters are made of thin (20 μm) constantan foil, their heat capacity is of the order of 40-50 J/(m•°K), and the effect of the heat capacity of the heater has to be taken into account already for materials with $\lambda < 0.15$ (see Fig. 1).

The traditional approach for methods based on unidimensional solutions consists in selecting such a value of heat capacity of the heater which guarantees a previously specified error of deviation of the corresponding problem, taking the heat capacity of the heater into account, from the solution that does not take the effect of heat capacity into account. However, it is not always possible to reduce the heat capacity of the heater to the required values (because of the difficulty of ensuring low contact resistance in conducting current to the film heaters, the impossibility of spray-coating highly resistive alloys with low thermal resistivity, because of the very weak currents that can be sustained by very fine heaters). This problem is posed particularly sharply in the investigation of heat-insulating materials with $\lambda < 0.05$.

The present article suggests another approach based on some regularities of the formation of a temperature field in a semibounded specimen with the surface heat capacity of the heater taken into account.

If we put in expression (7) $r_0 \to \infty$ (i.e., we deal with a unidimensional problem), we can rewrite expression (7) in the form

$$\Delta T_{\text{one}}(s) = \frac{w_0}{bs\sqrt{s} + c_h s^2}. \tag{9}$$

It is not difficult to find the original from the Laplace transform (9). In that case the surface temperature of the specimen is described by the following expression:

$$\Delta T_{\text{one}}(\tau) = \frac{2w_0\sqrt{\tau}}{b\sqrt{\pi}}\left\{1 - \frac{\sqrt{\pi}c_h}{2b\sqrt{\tau}}\left[1 - \exp\left(\frac{b^2\tau}{c_h^2}\right)\operatorname{erfc}\left(\frac{b\sqrt{\tau}}{c_h}\right)\right]\right\}. \tag{10}$$

An analogous expression was obtained in [7].

For convenience of analysis and calculation, expression (10) may be written as an expansion into a series:

$$\Delta T_{\text{one}}(\tau) = \frac{2w_0\sqrt{\tau}}{b\sqrt{\pi}} - \frac{w_0 c_h}{b^2} + \frac{w_0 c_h}{b^2}\frac{1}{\sqrt{\pi}}\left[\frac{1}{\frac{b}{c_h}\sqrt{\tau}} - \frac{1}{2\left(\frac{b}{c_h}\sqrt{\tau}\right)^3} + \frac{3}{4\left(\frac{b}{c_h}\sqrt{\tau}\right)^5} - \frac{8}{15\left(\frac{b}{c_h}\sqrt{\tau}\right)^7} + \dots\right]. \tag{11}$$

It follows from (11) that when $\tau \to \infty$, the temperature curve taking the heat capacity into account runs parallelly to the temperature curve disregarding the heat capacity and remains at the distance $w_0 c_h/b^2$ from it.

Since up to the numbers Fo ≤ 0.1 the temperature at the center of the circular heating spot with an error of less than 1% coincides with the unidimensional solution of [6], expression (11) may also be used for the case with a circular heater of constant power at the initial stage (up to Fo ≥ 0.1) of heat exchange. The radius of the heater r_0 and consequently also the duration of the unidimensional stage of the experiment (see the method suggested in [6]) are chosen by proceeding from the time of the experiment (for which the temperature curve with a previously specified error (e.g., less than 1%) runs parallelly to the temperatue curve disregarding the effect of the heat capacity of the heater) and also proceeding from the ratio of the thermal activity of the investigated material to the surface heat capacity of the heater.

If at this unidimensional stage (when the real temperature curve begins to rise practically parallelly to the temperature curve disregarding the effect of the heat capacity) we ascertain the points of the experimental temperature curve, and then smooth the obtained data by the least squares method by a straight line of the form $y = mx - n$, where $y = \Delta T(\tau)$; $x = \sqrt{\tau}$; $m = 2w_0/b\sqrt{\pi}$; $n = w_0 c_h/b^2$, we can determine the coefficient of thermal activity of the investigated material (proceeding from the coefficient m) and the lag of the temperature curve n. The time shift of the temperature curve τ' (see Fig. 1), proceeding from (11), is equal to $\sqrt{\tau'} = c_h\sqrt{\pi}/2b$ or $\sqrt{\tau'} = n/m$.

In accordance with the method of determining thermophysical characteristics suggested in [6], thermal diffusivity is determined at the two-dimensional stage of the experiment (with Fo > 0.2). It is calculated by the formulas presented in [6], with the difference that the points of the experimental temperature curve, ascertained at the two-dimensional stage of the experiment, have to be referred to the instant $\tau_e = (\sqrt{\tau}-\sqrt{\tau'})^2$, where τ is the running time of the experiment. Thus the experimental temperature curve shifts to the left on the axis $\sqrt{\tau}$ by the magnitude $\sqrt{\tau'}$. The calculation data presented in Table 1 show that the error of noncoincidence of the superposed temperature curves for Fo > 0.1 amounts to less than ±1%, which lies within the limits of the error of numerical transformation.

The values of dimensionless temperature presented in Table 1 were obtained by proceeding from the expression

$$\Theta(\mathrm{Fo}_1) = 2\sqrt{\mathrm{Fo}_1}\left(\frac{1}{\sqrt{\pi}} - \operatorname{ierfc}\frac{1}{2\sqrt{\mathrm{Fo}_1}}\right).$$

The dimensionless temperature $\Theta_{ch}(\mathrm{Fo}_2)$ was calculated by subtracting the result of the numerical conversion of the Laplace transform

$$\Theta_2(s) = \frac{\frac{1}{s}\,[1-\exp(-\sqrt{s})]^2}{\frac{r_0 b^2}{c_h\lambda} + \sqrt{s}\,[1-\exp(-\sqrt{s})]},$$

referred to the instant $\mathrm{Fo}_2 = \mathrm{Fo}_1 + \mathrm{Fo}' + 2\sqrt{\mathrm{Fo}_1\mathrm{Fo}'}$, from $\Theta(\mathrm{Fo}_2)$, where

$$\mathrm{Fo}' = \frac{a\tau'}{r_0^2} = \frac{a c_h^2 \pi}{4b^2 r_0^2}.$$

The error from their noncoincidence is $\delta = \dfrac{\Theta(\mathrm{Fo}_1) - \Theta_{ch}(\mathrm{Fo}_2)}{\Theta(\mathrm{Fo}_1)}\cdot 100\%$.

To verify the suggested method of correction of the data of temperature measurements, we made a heater with radius 9.9 mm from constantan wire with 0.1 mm diameter wound bifilarly turn by turn from the center of the heater. To make it more rigid, glue BF was poured over the heater. At the center of the heater a flat Chromel–Copel thermocouple was glued to it. The heater was situated between two specimens of polymethylmethacrylate ($\rho = 1174$ kg/m^3). The entire system consisting of the specimens and the heater was placed in an adjustable clamp. In the process of the experiment we registered the temperature–time dependence of the temperature increase at the center of the heater relative to the initial temperature. As signal amplifier of the differential thermocouple we used an amplifier F7024S/1, as

TABLE 1. Values of the Dimensionless Temperature of the Center of the Circular Heat Source Disregarding the Effect of Heat Capacity Θ (Fo_1) and Taking the Effect of the Heat Capacity of the Heater into Account Θ_{ch} (Fo_2) and of Percentages δ of Their Noncoincidence for Polymethyl Methacrylate and Rubber

Fo_1	Θ	Fo_2	Θ_{ch}	δ, %
Polymethyl methacrylate $\rho=1150$ kg/m³, $a=1{,}29\cdot10^{-7}\frac{m^2}{sec}$, $\lambda=0{,}162$ W/(m·°K) [9], $r_0=10\cdot10^{-3}$ m, $c_h=500$ J/(m²·°K)				
0,08	0,3176	0,1012	0,3194	—0,57
0,09	0,3359	0,1124	0,3388	—0,86
0,1	0,3529	0,1236	0,3542	—0,37
0,15	0,4224	0,1786	0,4234	—0,24
0,2	0,4739	0,2328	0,4715	0,5
0,25	0,5139	0,2865	0,5101	0,74
0,3	0,5462	0,3399	0,5465	—0,05
0,35	0,5728	0,3930	0,5705	0,4
0,4	0,5952	0,4459	0,5949	0,05
0,5	0,6313	0,5511	0,6312	0,02
0,6	0,6592	0,6559	0,6615	—0,35
0,7	0,6816	0,7603	0,6799	0,25
0,8	0,7001	0,8644	0,7025	—0,34
0,9	0,7157	0,9682	0,7177	—0,28
1,0	0,7291	1,072	0,7295	—0,05
Rubber $\rho=160$ kg/m³, $a=0{,}138\cdot10^{-6}\frac{m^2}{sec}$, $\lambda=0{,}05$ W/(m·°K) [9], $r_0=10\cdot10^{-3}$ m, $c_h=50$ J/(m²·°K)				
0,1	0,3529	0,1079	0,3530	—0,03
0,15	0,4224	0,1596	0,4222	0,05
0,2	0,4739	0,2111	0,4735	0,08
0,25	0,5139	0,2624	0,5131	0,16
0,3	0,5462	0,3135	0,5456	0,11
0,35	0,5728	0,3646	0,5721	0,12
0,4	0,5952	0,4156	0,5951	0,02
0,5	0,6313	0,5174	0,6305	0,13
0,6	0,6592	0,6191	0,6585	0,11
0,7	0,6816	0,7206	0,6816	0
0,8	0,7001	0,8220	0,7003	—0,03
0,9	0,7157	0,9234	0,7158	—0,01
1,0	0,7291	1,025	0,7291	0
Rubber $\rho=160$ kg/m³, $a=0{,}138\cdot10^{-6}\frac{m^2}{sec}$, $\lambda=0{,}05$ W/(m·°K) [9], $r_0=10\cdot10^{-3}$ m, $c_h=200$ J/(m²·°K)				
0,08	0,3176	0,1101	0,3198	—0,07
0,09	0,3359	0,1217	0,3396	—1,1
0,1	0,3529	0,1333	0,3538	—0,26
0,15	0,4224	0,1903	0,4231	—0,17
0,2	0,4739	0,2461	0,4767	—0,59
0,25	0,5139	0,3013	0,5111	0,54
0,3	0,5462	0,3560	0,5489	—0,49
0,35	0,5728	0,4103	0,5712	0,28
0,4	0,5952	0,4643	0,5975	—0,39
0,5	0,6313	0,5716	0,6321	—0,13
0,6	0,6592	0,6782	0,6622	—0,46
0,7	0,6816	0,7843	0,6823	—0,10
0,8	0,7001	0,8899	0,7038	—0,53
0,9	0,7157	0,9952	0,7164	—0,09
1,0	0,7291	1,100	0,7299	—0,11

measuring instrument a universal voltmeter V7-23. The experimental thermogram is shown in Fig. 2. From the experimental thermogram we determined the surface heat capacity of the heater (c_h = 398 J/(m^2•°K)), and by the suggested method we calculated the thermal activity and thermal diffusivity (b = 588 W•sec$^{1/2}$(m$_2$•°K), a= 1.28•10^{-7} m^2/sec, λ = 0.21 W/(m•°K)). We note that according to the data of [8] for T = 290°K for polymethyl metacrylate with ρ = 1170 kg/m^3, λ = 0.198 W/(m•°K), a = 1.18•10^{-7} m^2/sec. The obtained data were substituted into expression (7). It can be seen from Fig. 2 that the experimental curve agrees well with the results of the numerical conversion of this expression.

NOTATION

r, z, coordinates; τ, time; r_0, w_0, c_h, radius, specific power, and surface heat capacity, respectively, of the heater; a, b, λ, ρ, thermal diffusivity, thermal activity, thermal conductivity, and density, respectively, of the investigated material; $\Delta T(\tau)$, $T(r, z, \tau)$, T_0, excess temperature of the center of the heater, temperature of the investigated specimen, initial temperature of the specimen, respectively; ierfc(x), multiple probability integral; erfc(x), additional probability integral; τ', time-dependent shift of the temperature curve; $\Theta(Fo_1)$, $\Theta(Fo_2)$, dimensionless temperatures of the center of the central heat source disregarding the effect of the heat capacity of the heater referred to the instants Fo_1 and Fo_2, respectively; $\Theta_{ch}(Fo_2)$, dimensionless temperature of the center of the heater taking time shift into account.

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